lutions of salicylic, boracic, and similar acids, naphthalene and the metallic chlorides, separately and in combination, and in various strengths of alcohol and glycerine.

With these fluids, tests are being made by immersing insect larvæ and imagines, of all of which specimens careful notes respecting color and measurements are made before immersion in the fluids. Inspection is made frequently, and any changes in the specimen are noted. Especially valuable results are expected from simultaneous experiments on a large number of larvæ of the same species of lepidoptera. For example, a number of larvæ of an Arctian moth, all taken at one time and all of nearly equal age, introduced into a series of different fluids, show especially well by comparison the effects of the various preservative agents.

The experimenting was begun too recently to allow of any collaboration of results thus early. At a future meeting of the Academy, the experimenters hope to present the result of their work.

IMAGINARY FOCAL PROPERTIES OF CONICS.

BY H. B. NEWSON, LAWRENCE.

In modern geometry, the foci of a curve are defined as the points of intersection of the tangents to the curve from the imaginary circular points at infinity. Since four such tangents can be drawn to a conic, and since four lines intersect in six points, it would seem that every conic has six foci. But the circular points at infinity count for two; so there are only four finite foci, two real, and two imaginary. Many authors refer to the existence of the imaginary foci. C. Smith, in his conic sections, page 205, has shown that the imaginary foci are a pair of conjugate imaginary points, situated on the minor axis equi-distant from the center. Elsewhere (Annals of Math., vol. V., page 1) I have called attention to the fact that imaginary focal properties of conics may be developed which are the counterparts of the real focal properties.

I propose in this paper to set down in parallel columns the real and imaginary focal properties of conics, so that the reader can see at a glance what theorems concerning imaginary foci correspond to known theorems concerning real foci. I shall give first the theorems for the ellipse. They are given without proof. The simplest method of proof for the imaginary properties is to change a into b, and x into y, in the ordinary demonstration of the real properties.

REAL.	IMAGINARY.
The sum of the distances of any point on an ellipse from the real foci is constant, and equal to $2a$.	The sum of the distances of any point on an ellipse from the imaginary foci is constant, and equal to $2b$.
The real eccentricity is $e=\sqrt{\frac{a^2-b^2}{a^2}}$.	The imaginary eccentricity is $\acute{e}=\sqrt{\frac{b^2-a^2}{b^2}}$.
The distance of either real focus from the center is ae.	The distance of either imaginary focus from the center is $b\acute{e}$.
The length of the real semi-latus rectum is $a(1-e^2)$.	The length of the imaginary semi-latus rectum is $b(1-\acute{e}^2)$.
The lengths of the real focal radii are $a\pm ex$.	The lengths of the imaginary focal radii are $b\pm\acute{e}y$.

The polar equation of an ellipse, a real focus being a pole, is

$$r=\frac{a(1-e^2)}{1\pm e\cos.\theta}.$$

The locus of the foot of the perpendicular from either real focus on a tangent is the major auxiliary circle.

The rectangle contained by the perpendiculars from the real foci on a tangent is constant, and equal to b^2.

The real focal radii to any point on an ellipse make equal angles with the tangent at that point.

The major axis of an ellipse is divided by a real focus into segments, such that their product is equal to b^2.

The distance from the center to where the normal cuts the axis of x, or the subnormal, is e^2x^1.

A circle of radius a described from the extremity of the conjugate axis cuts the transverse axis in the real foci.

The polar of either focus is a directrix.

The distance from the center to either real directrix is $\frac{a}{e}$.

The distance of any point on an ellipse from a real focus is in a constant ratio to its distance from the corresponding directrix; this ratio is equal to the real eccentricity of the conic.

The equation of the tangent at the extremity of the real latus rectum is $y+ex=a$.

The equation of the normal at the same point is $y-\frac{x}{e}+ae^2=0$.

The circle described on a real focal radius as a diameter touches the auxiliary circle.

The polar equation of an ellipse, an imaginary focus being the pole, is

$$r=\frac{b(1-\acute{e}^2)}{1\pm\acute{e}\sin.\theta}.$$

The locus of the foot of the perpendicular from either imaginary focus on a tangent is the minor auxiliary circle.

The rectangle contained by the perpendiculars from the imaginary foci on a tangent is constant, and equal to a^2.

The imaginary focal radii to any point on an ellipse make equal angles with the tangent at that point.

The minor axis of an ellipse is divided by an imaginary focus into segments, whose product is equal to a^2.

The distance from the center to where the normal cuts the axis of y, or the conjugate subnormal, is $\acute{e}^2y^1$.

A circle of radius b described from the extremity of the transverse axis cuts the conjugate axis in the imaginary foci.

The polar of an imaginary focus is an imaginary directrix.

The distance from the center to either imaginary directrix is $\frac{b}{\acute{e}}$.

The distance of any point on an ellipse from an imaginary focus is in a constant ratio to its distance from the corresponding directrix; this ratio is equal to the imaginary eccentricity of the conic.

The equation of the tangent at the extremity of the imaginary latus rectum is $x+\acute{e}y=b$.

The equation of the normal at the same point is $x-\frac{y}{\acute{e}}+b\acute{e}^2=0$.

The circle described on an imaginary focal radius as a diameter touches the minor auxiliary circle.

Other theorems in respect to the ellipse might be added; but enough have been given to illustrate the theory. The above list contains the most of the important propositions.

If the conic be a hyperbola, two of the foci are still real, and two imaginary; but the imaginary foci coincide in position with the real foci of the conjugate hyperbola. The theorems concerning the imaginary focal properties of the hyperbola are almost all identical with the theorems concerning the real focal properties of the conjugate hyperbola. For example, the locus of the foot of a perpendicular, from an imaginary focus on a tangent, is the minor auxiliary circle; and the locus of the foot of a perpendicular, from a real focus of the conjugate hyperbola on a

tangent to the conjugate hyperbola, is the same circle. The whole theory for the hyperbola is, therefore, at once evident, and further details may be omitted.

The above theory for the ellipse is also capable of another interpretation. We may say that the conjugate ellipse is wholly imaginary, and that the theorems in the above list give us the focal properties of the conjugate ellipse.

In the case of the parabola, the theory of the imaginary foci is more complicated. Since the line at infinity is tangent to the parabola, two of the four tangents drawn from I and J coincide, and there are left only three tangents and three points of intersection. Two of these are I and J, and the other is the real finite focus. Thus the circular points at infinity are the imaginary foci of the parobola.

It would be interesting to know what properties the points I and J have in common with the finite foci of a conic, and especially with the finite focus of a parabola; but this question I have not investigated.

NOTES ON SOME SUMMER BIRDS OF ESTES PARK, COLORADO.

BY VERNON L. KELLOGG, LAWRENCE.

Estes Park is a beautiful little valley at the base of Long's Peak, about seventy miles northwest of Denver, being in latitude 40°24′ north, longitude 105°36′ west. The Big Thompson, affluent of the North Platte, fed by the melting snow of the Front Range, of which Long's Peak and adjacent peaks are a spur, winds through the six miles of the Park's extent, and its many small tributary streams come tumbling out of the surrounding hills and mountains in well-worn gorges, or glide gently down woods-lined valleys. Spreading away from the Big Thompson on either side, for half a mile or more, extends a luxuriant pasture. The Park narrows at its head, and the bounding lateral moraines, covered with ice-worn boulders, approach rapidly, until the Park is no park, but a gorge, which cuts deeply into the great snow-covered range, and is yet as wild and primeval in its aspect and condition as before the first camp-fire smoke ever drifted up from the now fairly civilized meadows along the Big Thompson.

The altitude of the Park is about 8,000 feet, and of the range from which its massive boulders have been brought about 13,000 feet; while Long's Peak rears its square-capped head to an elevation of 14,271 feet. The peaks and chasms of the range are white with never-melting snow, and in certain places—two, at least—the snow has become so compacted, and the well-marked crevasses and half-proven movements so sure evidences, that Hallet's and Chapin's Glaciers are beginning to attract more than local attention. Great tracts on the range sides are covered with spruce forests, but so many fires have swept over the region that the hills and valleys are covered with the charred spars of spruce and pine, some standing, others prostrate and entangled in the up-springing new growth.

The following notes on the avian fauna of the Park are from observations made during the summers of 1886–1889. A brief visit in June of this year (1890) gave opportunity for some corroborative observation:

1.—Anas strepera Linn. Gadwall.

A pair seen on a small pond in an offshoot of Estes Park, called Horseshoe Park. This duck is not an uncommon summer resident.

2.—Actitis maculata (Linn.) Spotted Sandpiper.

Not uncommon. A few seen along the Big Thompson, and a pair seen August 19 at a small pond near timber-line.